

# Continuum Mechanics II

Course Codes: MEG 416, SSG 431

Instructors: OA Fakinlede, O Adewumi & F George

# Topics to be Covered

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KINEMATICS

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THEORY OF STRESS & ENERGY FLOW

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NATURAL BALANCE LAWS OF MASS, LINEAR MOMENTUM, ANGULAR MOMENTUM

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THE PRINCIPLE OF ENTROPY AND ENERGY QUALITY

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PRACTICUM: MODELING, SIMULATION, FAILURE ANALYSIS & DESIGN

# Introduction



Syllabus at 300 Level was ambitious. Most of the topics beyond Tensor Analysis will be covered this term. <sup>3</sup>



Much of the material for 500 level were fully covered in 300 level and after this term, Students should be ready for their Capstone Projects using the Design Principles, Methods and Skills learned this term.



We need two hours for classroom, two design Laboratory hours and one hour for tutorial

# Requirements

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- If you scored lower than 65% in 300 level, do yourself a favour, go practice the work on Vectors and Tensors so that you do not pay a second penalty for your mediocrity.
- Fusion 360 License and active use for Simulations, Design and Analysis
- Mathematica. Symbolics Algebra, Graphics & Computations.
- Textbook: OA Fakinlede, Continuum Mechanics for Modeling, Simulation and Design
- Other course materials: Self test platform, Slides, videos etc.

# Software



Fusion 360 by Autodesk: For 3D Solid Modeling, Simulation, Animation, Analysis and Design



Mathematica by Wolfram Research: Mathematical Computations & Graphics



LaTeX manuals LaTeX Software (Mostly free on the web) These are needed to properly communicate with us on the web. Mr Ayoade Oriola [ayoade@s2pafrica.org](mailto:ayoade@s2pafrica.org) will intercept your postings on our site. He will guide you and help with your LaTeX. Please cooperate with him.

# Do Yourself a Favour!

- An Engineer that has the skills to Model, Design and Simulate in Fusion 360 will be good enough for any market, local and overseas. It is an insurance against joblessness.
- The same skills can help you get very good, non-trivial design projects in your Capstone year, that can change your life.
- Mathematica can help you program the computer easily to get engineering calculations done.
- Add this to the Continuum Mechanics training, a very good combo indeed!

# Work Plan

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- From Week 2, we shall have two hours of class teaching, two hours of design lab practice every week. We may add one week of tutorial if necessary.
- Design class will aim at improving your 3D Solid Modeling Capability so that you can create objects for simulation and prototyping.
- The regular classes will look like last term.

Item	Description	Percentage	Due
Test 1	Class Based Test	15	Week 5
Test 2	Class Based Test	15	Week 10
Self Tests	Selection from self tests	15	Week 13
Design Portfolio	<ol style="list-style-type: none"> <li>1. Models from your environment (5)</li> <li>2. Improve them and Simulate Loads, Constraints, Contacts, etc. (5)</li> <li>3. Optimize Design (5)</li> <li>4. Innovative (Creative) Design (5)</li> <li>5. Terminal Report (5)</li> </ol>	25	Week 4  Week 8 Week 13
Final Exam		30	Week 13
Web Participation	Bonus for asking or answering questions on the web. Commenting sensibly will also count.	5	Week 13

# Course Grading

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# What is Kinematics?

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- Kinematics deals with deformation and motion.
- It is the geometry of deformation when such deformations are invariant with time; it is the geometry of motion when the deformation is time dependent.
- It defines many terms we use in all of mechanics.
- It is very important to gain deep understanding of the geometry before doing anything else!

*The various possible types of motion  
leaving aside the causes to which  
the initiation of motion may be ascribed*  
*E. T. Whittaker*

# Kinematics

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- We are often impatient; we want to know about forces, stresses, inertia and other force fields.
- It is a grave mistake to run to kinetics without first mastering the Geometry of Shape Changes & Motion that constitute the Science of Kinematics.
- Kinematics provides for us the meaning of geometric concepts such as Linear Strain, Shear Strain, Displacement, Deformation Gradient, etc.

# Stress & Energy Flow

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You will be able to answer the question, “What is stress” in a non-trivial way! If you think stress is simply “Force per unit area”, it begs the questions: What force? What area? As area itself changes as a result of the application of a force!

Is stress a scalar, a vector or a tensor? Is it some or all of these? What is contact stress? What stress leads to material failure? How is this related to yield stress?

How do you quantify the rate energy flows into a material in contact with another or in the ambient environment?

# Mass, Linear & Angular Momentum

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- Recall that in courses such as Strength of Materials, Fluid and Solid Mechanics, Heat Transfer, etc., you are usually invoking natural laws to obtain relevant differential equations describing motion, etc. These include:
  1. Conservation of mass, momentum and angular momentum
  2. First and second laws of thermodynamics
- Here is where continuum mechanics shines: It decouples these balances from the materials; applies them once and for all and obtains equations that are valid for all materials in a concise tensor form.
  - These are then added to constitutive laws for each material to obtain their governing equations.

# Entropy Principle

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*“The law that entropy always increases holds, I think, the supreme position among the laws of Nature. If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations – then so much the worse for Maxwell's equations. If it is found to be contradicted by observation – well, these experimentalists do bungle things sometimes. But if your theory is found to be against the Second Law of Thermodynamics, I can give you no hope; there is nothing for it to collapse in deepest humiliation.”*

—AS Eddington

# Modeling Simulation Failure Analysis & Design

- These will be dealt with in our practical classes. If you don't take these seriously, your performance in the course will be mediocre at best.
- More importantly, you miss out on the real practical implications of all the effort. I think better things of you:
- You can use this as an opportunity to develop a number of skills that can define your professional life. These include
  - Solid Modeling in Fusion 360, Animation, Simulation, Creative Design, Mechanical Design

# Solid Modeling

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- 200 Level Course. If you have not done it well, here is another chance. We will go through with four or five examples from our environment
- Fusion 360 is the preferred software and we can assist you in it. Solid Works, Autodesk Inventor, Siemens Solid Edge or Pro Engineer are some possible equivalents.
- Armed with these, you should be able to make a technical replica of useful objects in YOUR environment.

# Creative Design

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- Armed with 3D modeling software, your creative design training may begin with modifying objects you have successfully turned into your own models. For example:
  - On the left is the picture of my reading lamp. Top right is a 3D Solid model of the same lamp in Fusion 360.
  - Inspired by the lamp and its model, the middle and lower RHS show a new lamp design using revolute joints of Fusion 360 Component Assembly
  - Creating an improved lamp that can more easily be packed and travelled with!



# Failure Analysis

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- Does it concern you that your engineering faculty and your university is littered with failed concrete benches? Did you even observe them?
- Is there anything you, as a student in Mechanical Design, can do about it? Are you as helpless as everyone else?
- Do you offer opinions on these and similar issues? What are the bases of such opinions? Are they related to any engineering computations?
- If so, you are doing failure analysis. If not, you are simply as ignorant as the rest!

# Failure Analysis

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The shed in front of Civil Engineering Department. These were supplied in 2005. Failure began as early as 2006



This was launched by Chairman of University Council, Dr Gamaliel Onosode in 2013. Failure started within five years.



These were more heavily built than the previous two. Three supports instead of two. Yet, the failures were more dramatic: DOA: Some were already failed structures even before the buildings of the DLI were commissioned!

# Failure Analysis

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- These were built sixty years ago - the year of Nigeria's independence from the British. They lasted 50 years before any significant failure. They are located between Systems Engineering and Metallurgical & Materials Engineering.
- Observe they are more slender than the others and are modular in construction.



# Failure Analysis: Posers

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- What led to the failures we have snatched from the mouth of victory? We already had working concrete benches before the others were constructed. Why did we get terrible benches?
- What really is the problem?
  - Design?
  - Construction?
  - Transportation?
  - Supervision?
  - Corruption?

# Failure Analysis: Engineers talk Quantitatively

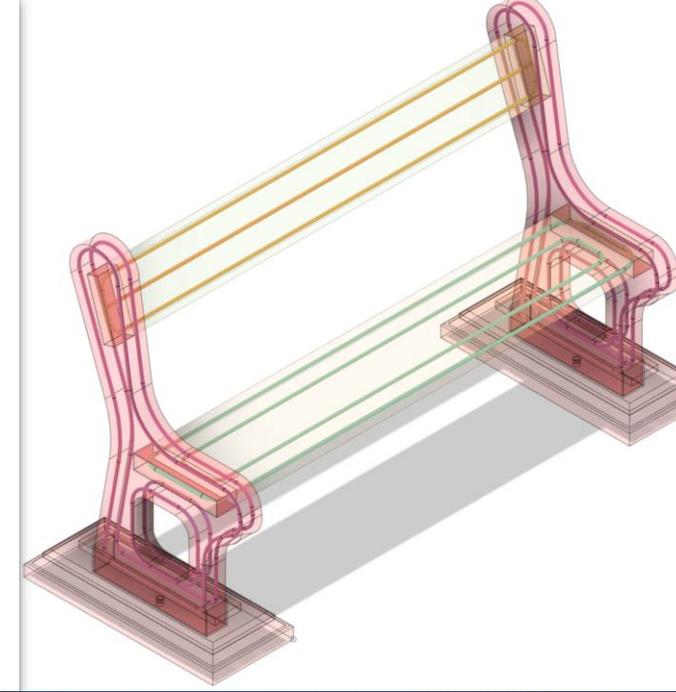
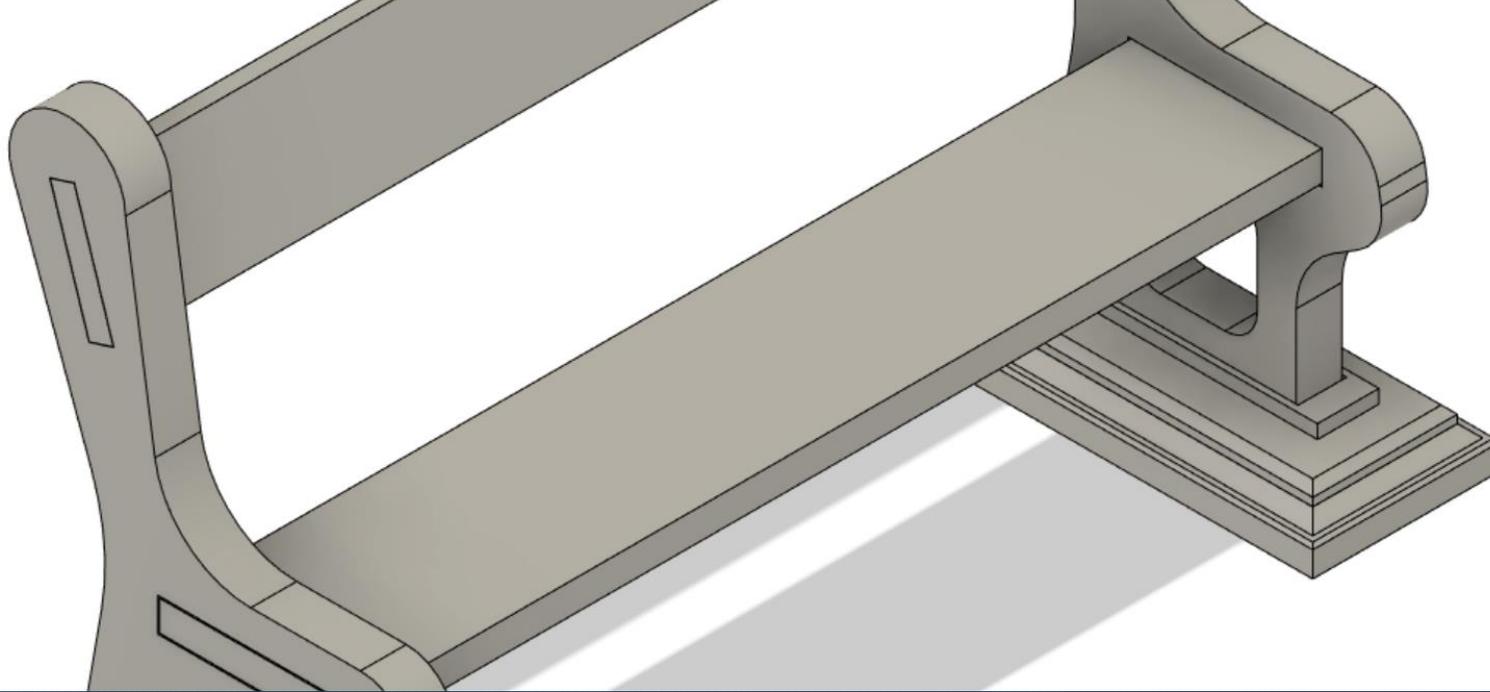
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- Observation: Everyone has an opinion on these failures:
  - Most of the opinions have no quantitative basis. Unfortunately, those opinions decide the next cause of action in many cases.
  - This lead to one failure succeeding another.
  - In most of the failures, the slabs were OK. Failures were at the supports
- Is it possible to quantitatively analyze these failures and come to superior, actionable solutions?
- Is it possible to improve on the concrete benches that were made sixty years ago and get better benches in 2020?

# Quantitative Answers: Simulation & Analyses

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- Begins with an accurate 3D Solid Model of a properly designed Component based Concrete Bench consisting of Two arms, Two supports, A slab and a back rest.
- Each of these is designed with proper reinforcement and tested with static loads.
- Scenario Analyses is coupled with simulation to first examine existing design, seek its improvement until optimality.
- These begin with the Solid model here:



## Solid Model of Designed Concrete Bench

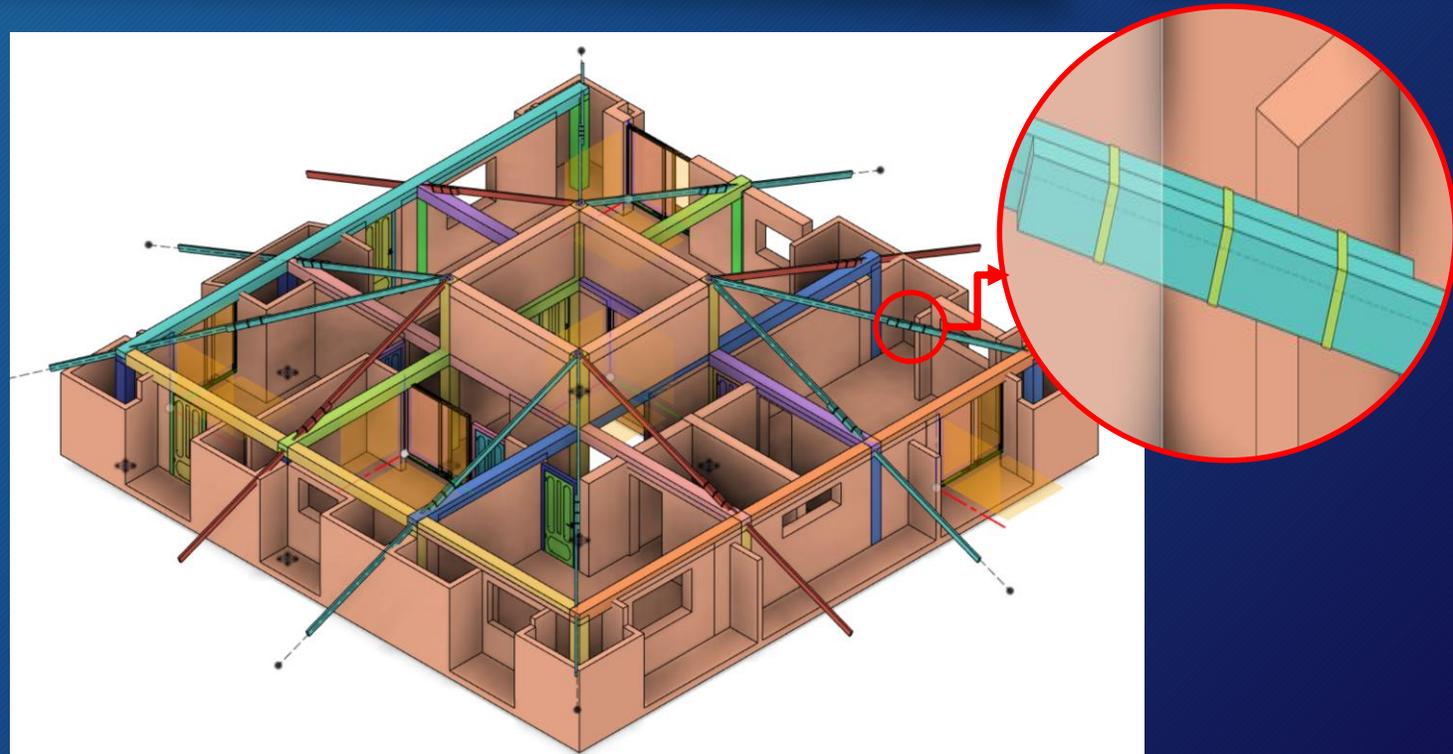
3D Solid model of the concrete bench with the second picture emphasizing the internal steel reinforcements. We have prepared this for simulation and optimization for failure analysis and design.

The reinforcements are exposed by the opacity control feature of fusion 360

# Failure Analysis: Two Wooden Beams

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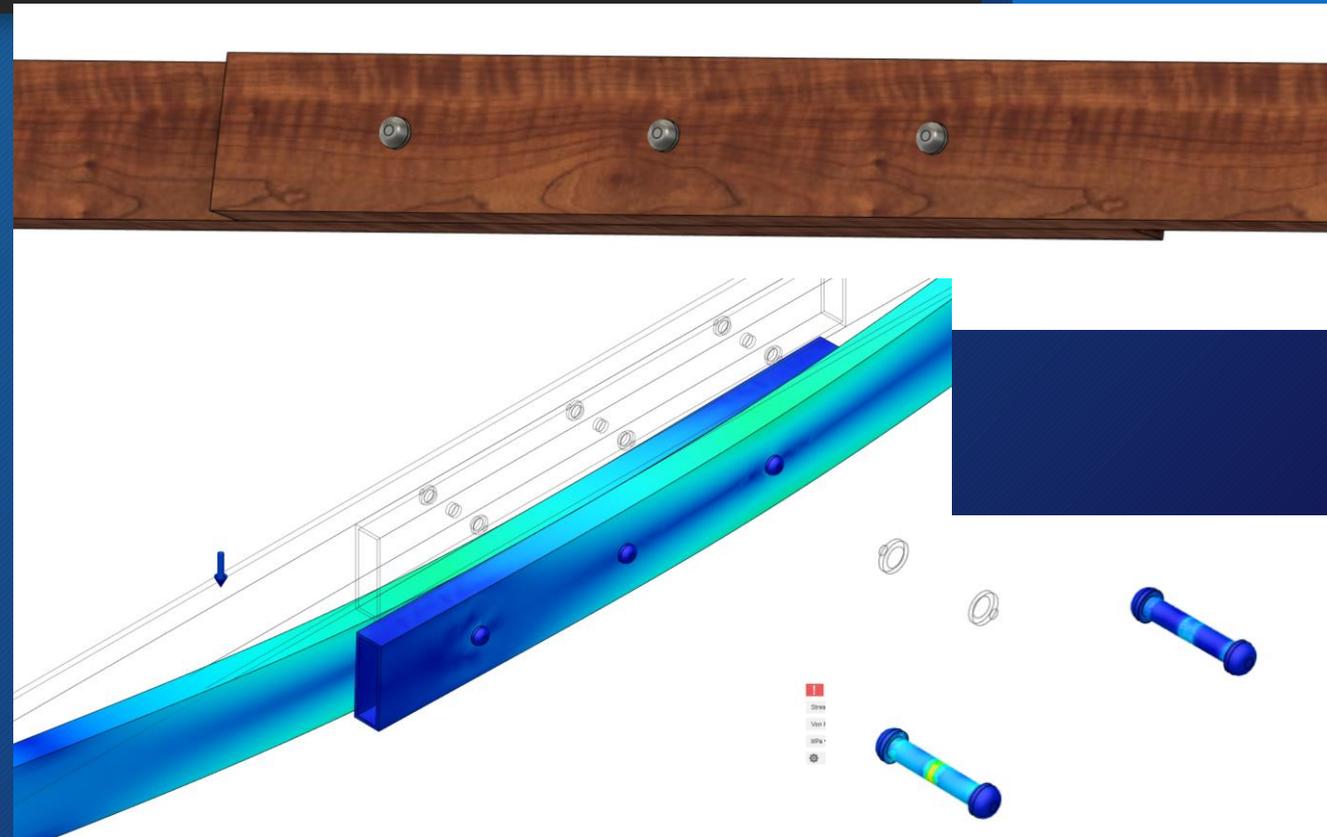
- On a construction site, the goal is to set up the trusses to roof a typical bungalow like the one shown here.
- The longest 2 by 6 structural wood sold in the market is 5 meters long. What happens when you need to span longer distances?
- Look at circled area:



# Wood Fasteners

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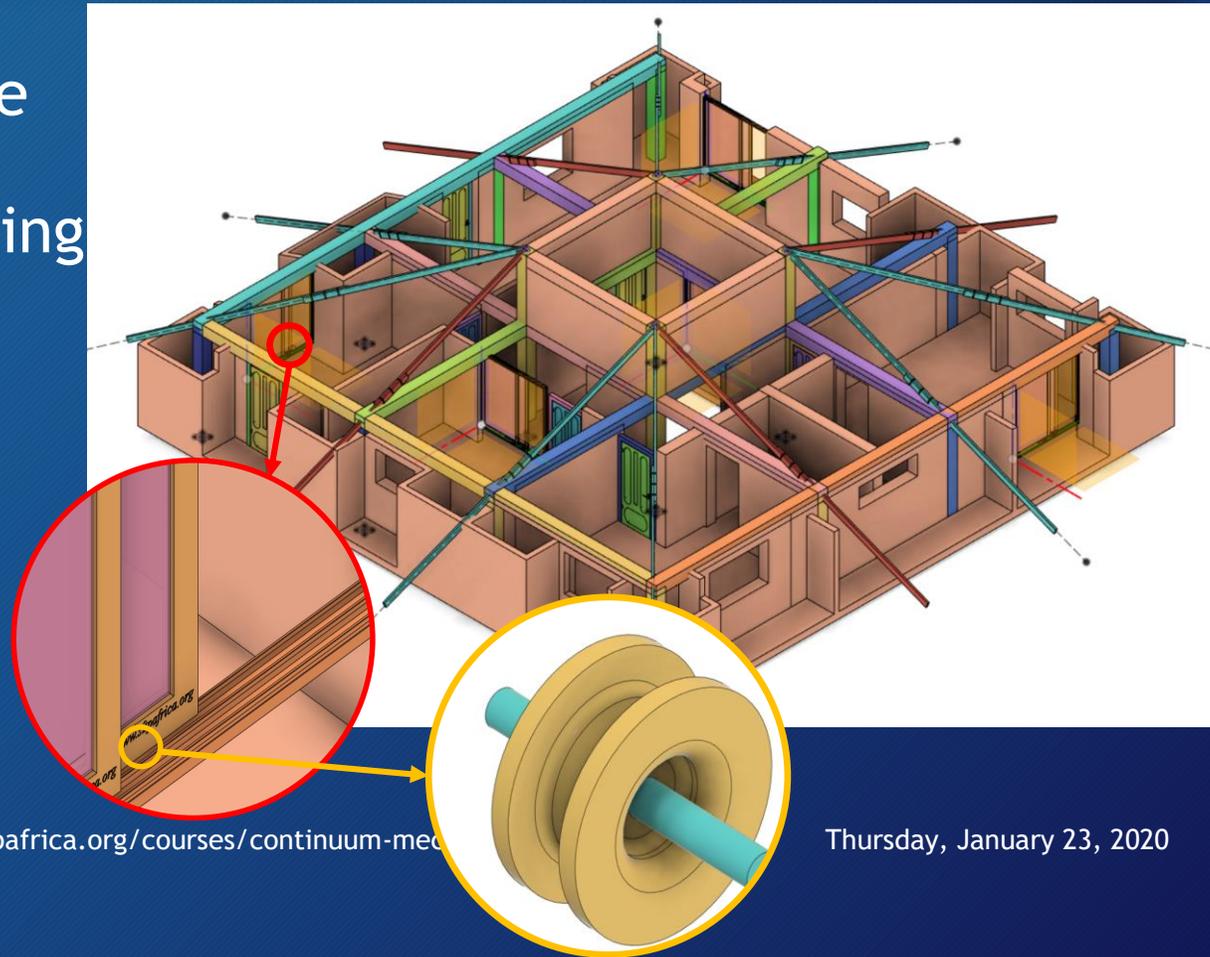
- In the example here, we chose to join the beams using bolts of a particular dimension arranged in uniform spacing.
- The simulation pictures are shown for the two beams and after removing the beams, we can emphasize the stresses on the bolts.
- Von Mises equivalent stresses are given as design criteria.



# Failure Analysis: Sliding Aluminum Doors

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- Or we are interested in the load borne by the roller assembly of aluminum sliding doors circled here:



# Failure as a result of displacement

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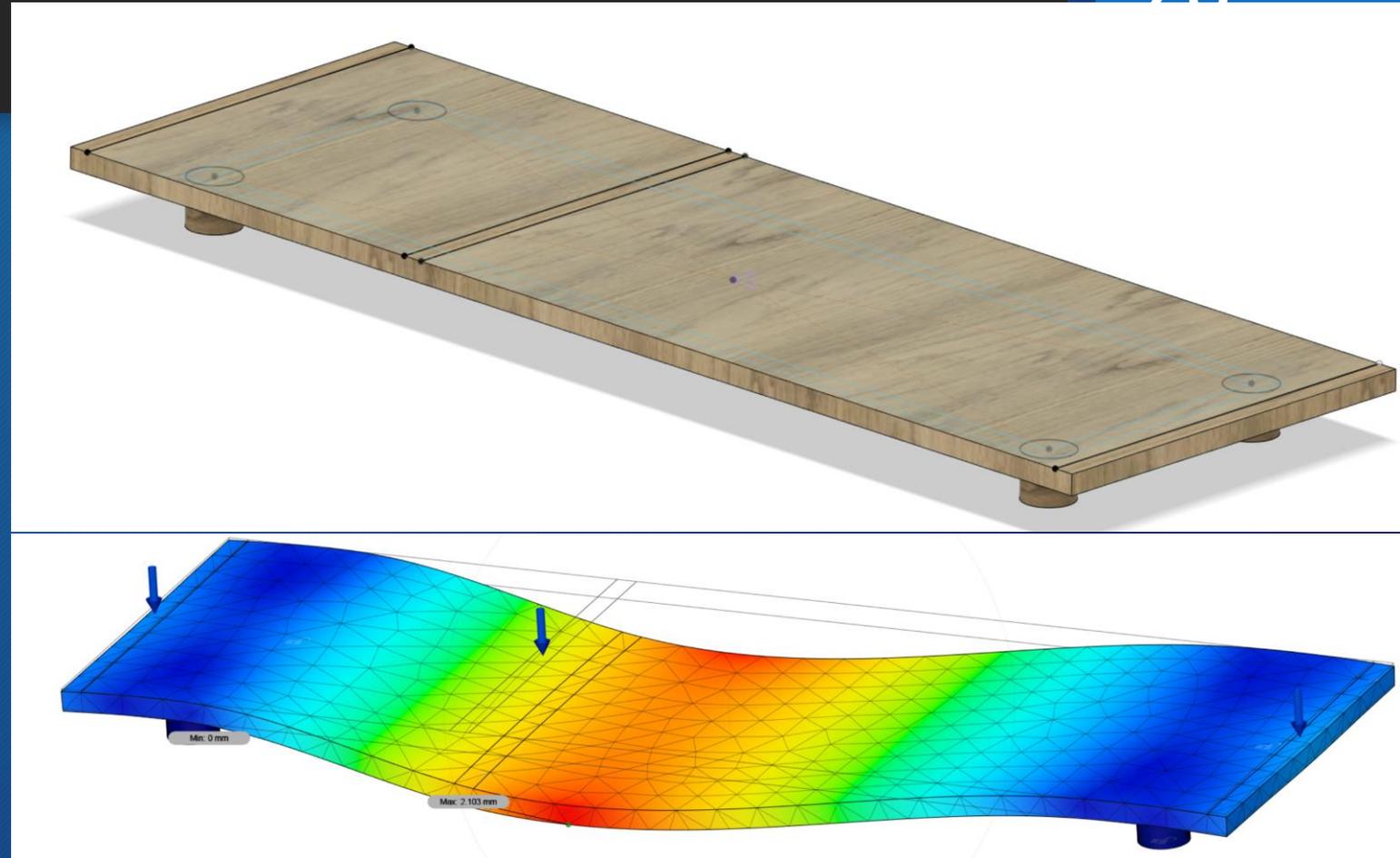
- An aircraft that has too much displacement on its wings upon landing might be lost in fire as a result. There are less dramatic cases where failure results from large displacements. Look at my drawer Cabinet as an example:



Observe that the drawers are closer at the top than they are at the lower part. Observe also that there are two metal supports on each side as shown in the photograph. We created the support structure in Fusion 360 as shown in the next slide and simulated the loading

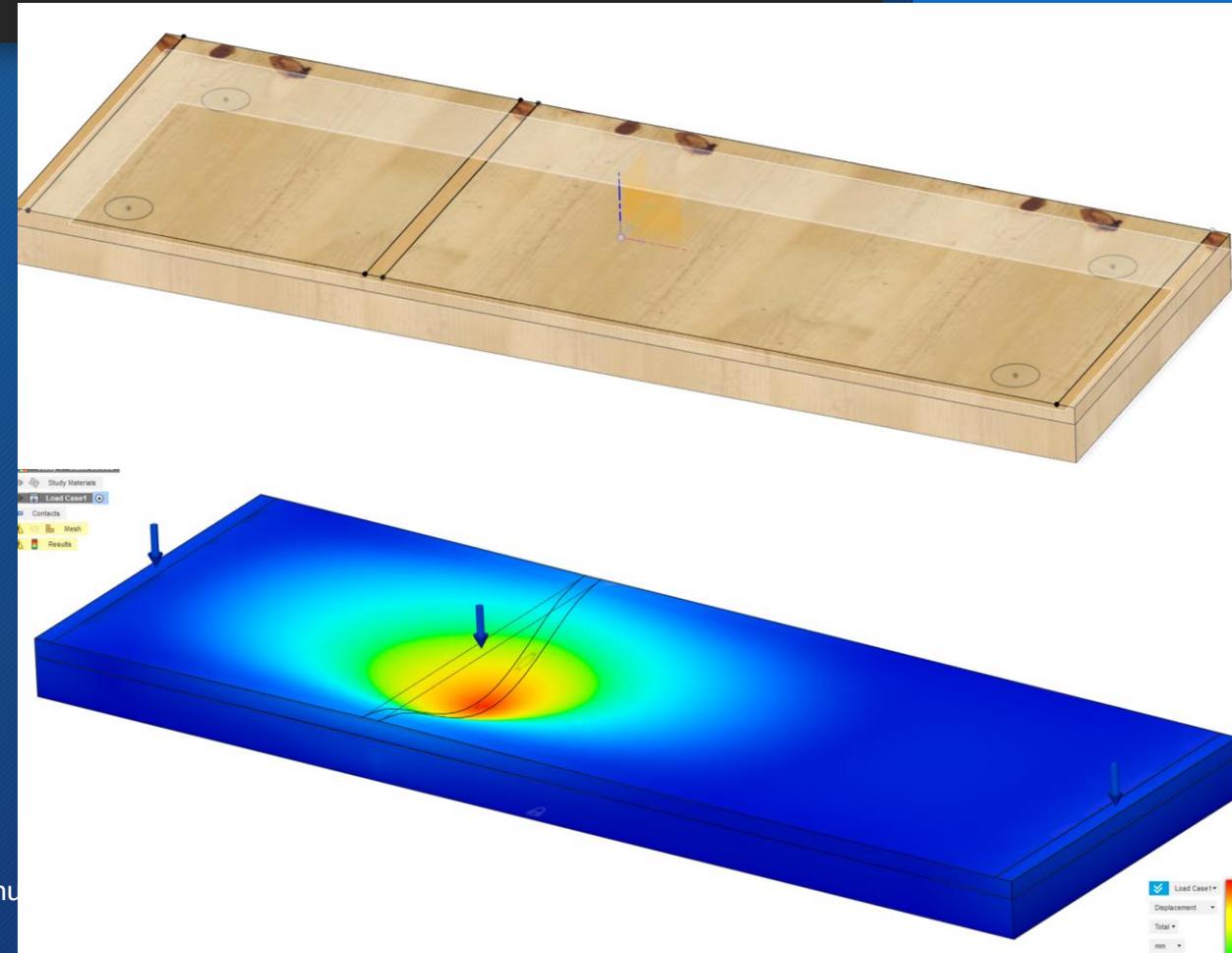
# Simulated Support

- Displacement Analysis of the support:
- Finite element analysis of the wooden beam and its support base is shown here. For a loading of 500 Newtons - a bag of cement at each position shown, we obtained a maximum displacement of 2.1 mm



# Improved Support

- The same base with a different support gives a maximum displacement of 0.1 mm which is about 1/20 th of the actual cabinet. Giving this kind of support could have obviated the failure inevitable starting from closing the gap between two parallel drawers and having them intercept each other, thereby making the cabinet fail.



The practical examples given emphasize geometry. We need a 3D solid model as the starting point of analysis.

Kinematics provides the accurate description of the geometry - leading us to precise concepts of deformation terms such as **displacement**, **strain**, and other descriptors of deformation. From there we can know about the rates of strain, stretching rates, vorticity and other descriptors of motion.

One of the most important triumphs of Kinematics is the separation of motions and displacements that do not matter to the study of material behavior. This is done in the **multiplicative decomposition** of the fundamental deformation descriptor: The Deformation Gradient.

# Theoretical Analysis: Everything Starts from Geometry

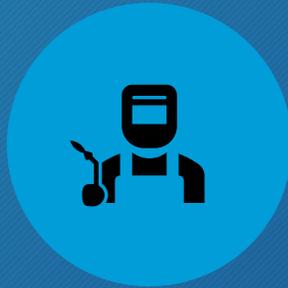
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# Leibniz- Reynolds Transport Theorem

1. Differentiation of integrals plays key role. **Leibniz Reynold's Transport** Theorem extends the familiar Leibniz theorem from calculus. A generalized version of this is provided for scalars, vectors and tensor valued fields.
2. Basic deformations such as rotation, translation, **extension, shear** and as well as constrained flow such as **Irrotational, Steady** and **Rigid** flow fields are discussed in this chapter.

# Designing for Stress, Strain and Displacement.

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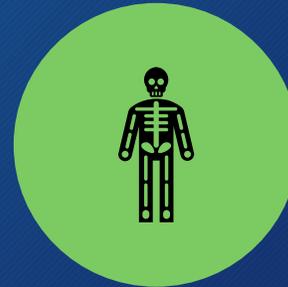
Engineers take great pain and spare no effort to ensure that service components do not undergo stresses or forces beyond their capability.



Displacements also must be kept with limits to ensure safe operation of certain equipment.



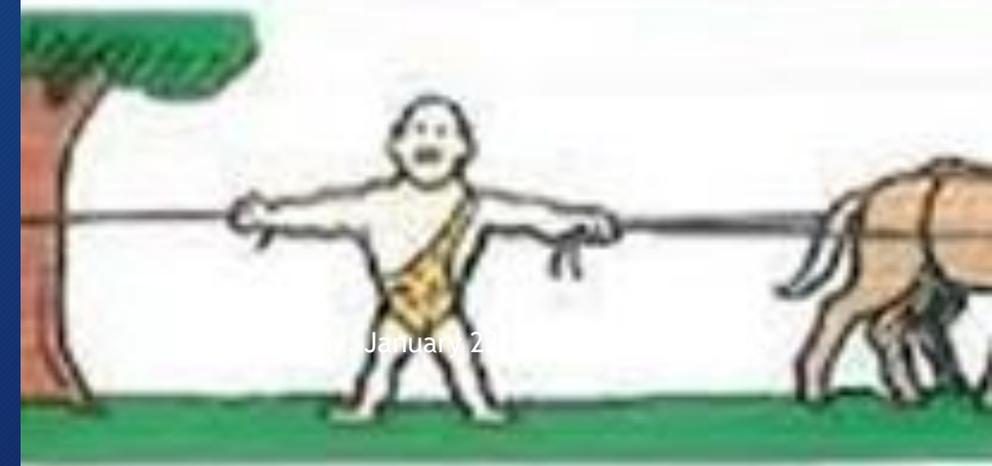
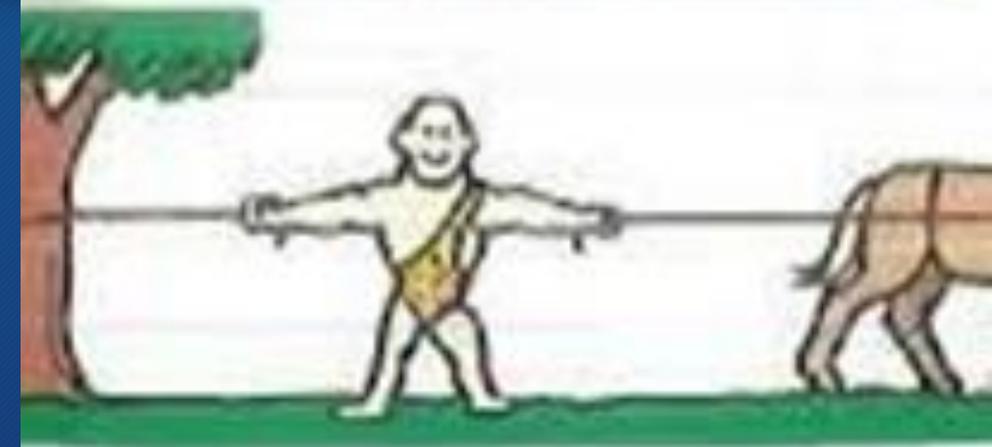
Mechanics of materials provide the science to make the necessary computations to ensure this. Everything we shall learn has a geometric background.



Loading concepts such as tension, compression, torsion, bending, etc., have clear geometric implications.

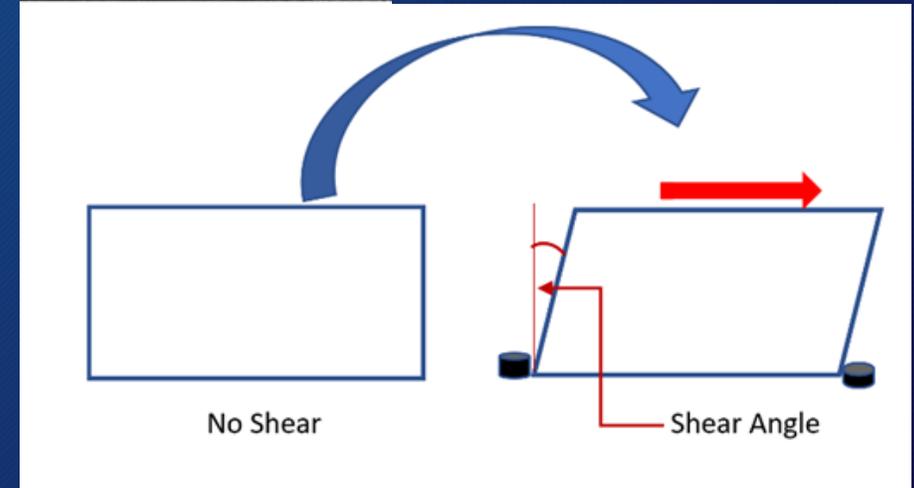
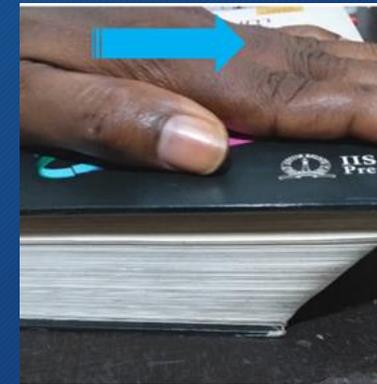
# Tension

- The unfortunate fellow in the picture below is obviously in tension. We have three examples of how that tension can be applied.
- In each case, the geometrical issue is the possible separation of the hands from the body. There are forces involved obviously, but their effects and consequences are about the attempt to change in his geometry: Separation of his hands or the ripping apart of his body! This is tension: It tends to lengthen.



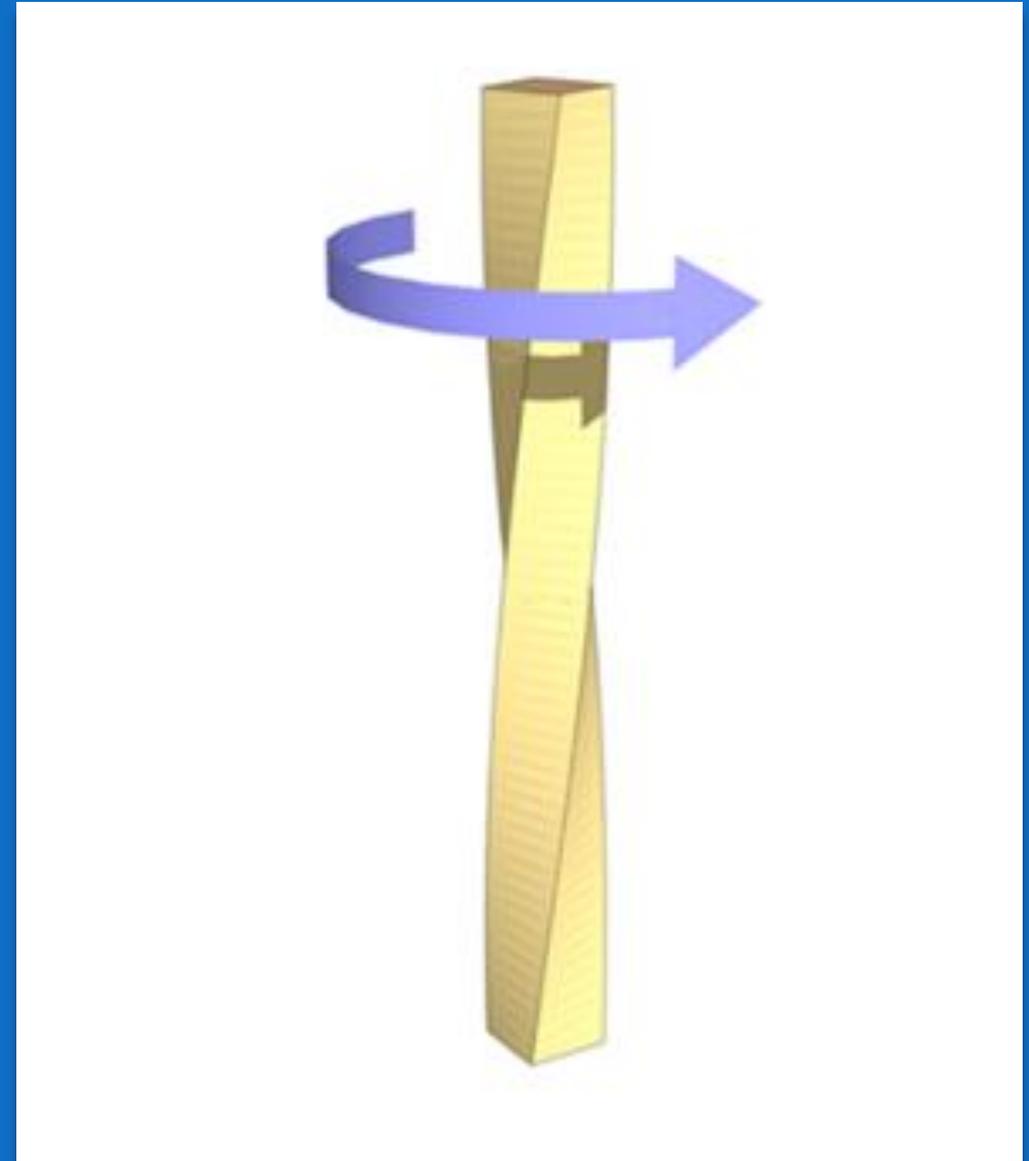
# Shear

- The geometry of compression is the opposite. Try to carry two or more bags of cement on your head, you will get that feeling that your height is being reduced.
- In the book shearing deformation diagram shown, the essential geometrical effect is the turning of the roughly rectangular cross section into a parallelogram. Decreasing an angle and increasing another. This kind of deformation and motion occurs all the time when we hoe, shave (sheep shearing, etc.,) or in a much cataclysmic way, in earthquakes, landslides and other less portentous ways. Shearing creates relative angular displacements and motions in parts of the material body of interest to us.



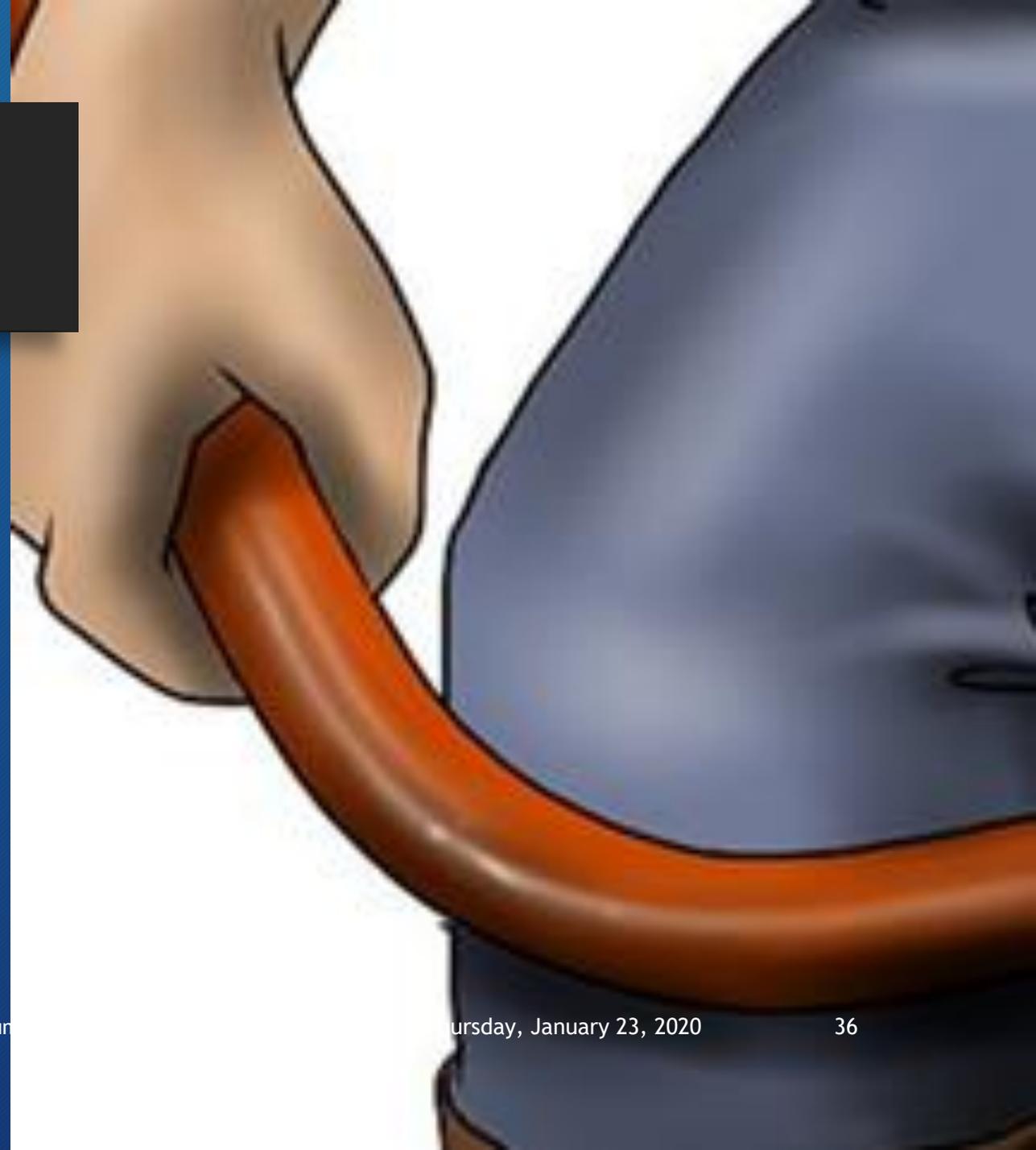
# Torsion

- Torsion also creates relative angular displacements. This is usually caused by moments in the longitudinal direction. In practical cases, it induces, in addition to the relative angular displacements, a warping deformation or motion in the longitudinal direction. Shafts, loaded by longitudinal moments, are often subjected to the shearing and warping caused by torsion.



# Bending

- Bending means, among other things, the alteration, or attempted alteration of the curvature of a body. In a straight bar, the curvature is decreased. Such changes in curvature also creates compression and tension on opposite sides of such bars. In a prismatic body, there is a cease-fire zone that is essentially neutral - free of the tension and compression, in a 3D body, there will usually be accompanying shear stresses. When the latter is NOT the case, it is said to be pure bending - the curvature alteration leading only to tensile and compressive stresses.



- While the geometry of motion leads to the definition of purely geometric quantities such as strain, stretch and related quantities, the foregoing show that the loading situations encountered can also be accurately described by the geometry.
- Many of the names are from the geometrical effects of the loads on the objects of concern. In shear, for example, the change from a rectangle - the reduction in the right angle, is the definition, as we shall see, of shear strain!

# Geometry & Loading

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# Thermal Stresses

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- The case of thermal stresses is interesting. Consider a bar compelled, as shown in figure shown to keep its length unchanged. Ordinarily, the heat supplied should lead to elongation. However, the geometrical constraint here creates the forces to prevent it from doing so. This causes thermal stresses.
- This bar will be in compression as the effect of geometry is essentially to create the forces decreasing its length from the state the applied heat would have placed it.



- Kinematics, the study of the geometry of deformation and motion “is the machinery for describing all possible deformations a body can undergo.”
- “Motion” itself is a time-dependent deformation.
- Kinematics is the application of geometry to these.

# Kinematics

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# Web Participation

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- Remember, there are five bonus marks at stake for participating on the Web.
- We shall be more strict this term. Your participation will be observed. It will be taken as a measure of your seriousness in the course.
- If you have questions on LaTeX: the language of mathematical communication on the web, contact [ayoade@s2pafrika.org](mailto:ayoade@s2pafrika.org)
- Those who do not improve their ability to properly communicate mathematical equations may find their questions go unanswered.

# Self Test

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- Self Tests, just as we had them last semester will count for 15% of the semester grades.
- They are designed to ensure that EVERY hardworking student, not only passes the course but get descent grades.
- Furthermore, the more you practice, the more understanding you gain. It is therefore not likely to continue to obtain good grades in self tests without it reflecting on your examination grades.