1. Given that $\forall\ \mathbf{v} \in \mathbf{V}$, $\boldsymbol{a} \cdot \mathbf{v} = \mathbf{b} \cdot \mathbf{v}$, Show that $\boldsymbol{a} = \mathbf{b}$

2. Given that $\forall\ \mathbf{v} \in \mathrm{V}$, $\boldsymbol{a} \times \mathbf{v} = \mathbf{b} \times \mathbf{v}$, show that $\boldsymbol{a} = \mathbf{b}$

3. Given that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors, find the values of scalars α and β in the equation, $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \boldsymbol{\alpha}\mathbf{u} + \boldsymbol{\beta}\mathbf{v}$

4. Given that $\mathbf{n}$ is a unit vector, use the fact that $\mathbf{n} \cdot \mathbf{u}$ is the projection of the vector $\mathbf{u}$ in the direction of $\mathbf{n}$ to represent $\mathbf{u}$ as $(\mathbf{n} \cdot \mathbf{u})\mathbf{n} + \mathbf{n} \times (\mathbf{u} \times \mathbf{n})$ or $(\mathbf{n} \otimes \mathbf{n})\mathbf{u} + \mathbf{n} \times (\mathbf{u} \times \mathbf{n})$.

5. Simplify the following by employing the substitution properties of the Kronecker delta $(a) e_{ijk}\delta_{kn}$, $(b) e_{ijk}\delta_{is}\delta_{jm}$ $(c) e_{ijk}\delta_{is}\delta_{jm}$ $(d)\ a_{ij}\delta_{in}$ $(e)\delta_{ij}\delta_{jn}$ $(f)\delta_{ij}\delta_{jn}\delta_{ni}$

   $(\boldsymbol{a})\ \boldsymbol{e}_{ijn}$ $(\boldsymbol{b})\boldsymbol{e}_{smk}$ $(\boldsymbol{c})\boldsymbol{e}_{smk}$ $(\boldsymbol{d})\boldsymbol{a}_{nj}(\boldsymbol{e})\boldsymbol{\delta}_{in}$ $(\boldsymbol{f})\boldsymbol{\delta}_{ij}\boldsymbol{\delta}_{ji} = \boldsymbol{\delta}_{ii} = \boldsymbol{\delta}_{11} + \boldsymbol{\delta}_{22} + \boldsymbol{\delta}_{33} = \mathbf{3}$

6. Show that the sum of triple products, $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} + (\mathbf{v} \times \mathbf{w}) \times \mathbf{u} + (\mathbf{w} \times \mathbf{u}) \times \mathbf{v} = \mathbf{o}$

7. Given that, $I_{ij} = \iiint_V \left(x^m x^m \delta_{ij} - x^i x^j\right) \rho(x^1, x^2, x^3) dx^1 dx^2 dx^3$ is the moment of inertia along thebaxis $i - j$ where $x = x^1, y = x^2, z = x^3$ and $\rho(x^1, x^2, x^3)$ is scalar density of the material find all the components of the tensor.

8. Show that the Cylindrical Polar basis vectors,

$$\boldsymbol{e}_r(r, \phi, z) = \cos\phi\, \boldsymbol{i} + \sin\phi\, \boldsymbol{j}$$
$$\boldsymbol{e}_\phi(r, \phi, z) = -\sin\phi\, \boldsymbol{i} + \cos\phi\, \boldsymbol{j}$$
$$\boldsymbol{e}_z(r, \phi, z) = \boldsymbol{k}$$

constitute an orthonormal system. [**Hint:** Show their magnitudes are unity and they are pairwise orthogonal].

9. Show that the Spherical Polar basis vectors

$$\boldsymbol{e}_\rho(\rho,\theta,\phi) = \sin\theta\cos\phi\,\boldsymbol{i} + \sin\theta\sin\phi\,\boldsymbol{j} + \cos\theta\,\boldsymbol{k}$$
$$\boldsymbol{e}_\theta(\rho,\theta,\phi) = \cos\theta\cos\phi\,\boldsymbol{i} + \cos\theta\sin\phi\,\boldsymbol{j} - \sin\theta\,\boldsymbol{k}$$
$$\boldsymbol{e}_\phi(\rho,\theta,\phi) = -\sin\phi\,\boldsymbol{i} + \cos\phi\,\boldsymbol{j}.$$

Constitute an orthonormal system. [**Hint:** Show their magnitudes are unity and they are pairwise orthogonal].

10. Find the derivatives of all the basis vectors in Q9 and Q10.

11. Given that the position vector in spherical coordinates is given by $\boldsymbol{R} = \rho\boldsymbol{e}_\rho(\theta,\phi)$, where $\boldsymbol{e}_\rho = \sin\theta\cos\phi\,\boldsymbol{i} + \sin\theta\sin\phi\,\boldsymbol{j} + \cos\theta\,\boldsymbol{k}$ show that the set $\left\{\frac{\partial\boldsymbol{R}}{\partial\rho},\frac{\partial\boldsymbol{R}}{\partial\theta},\frac{\partial\boldsymbol{R}}{\partial\phi}\right\}$ forms a basis set of orthogonal vectors. This is called the natural basis for the coordinate system. Normalize them to form an orthonormal (physical) basis.

12. Given that the position vector in cylindrical polar coordinates is given by $\boldsymbol{R} = r\boldsymbol{e}_r + z\boldsymbol{e}_z$, where $\boldsymbol{e}_r = \cos\phi\, \boldsymbol{i} + \sin\phi\, \boldsymbol{j}$, and $\boldsymbol{e}_z = \boldsymbol{k}$ show that the set $\left\{\frac{\partial \boldsymbol{R}}{\partial r}, \frac{\partial \boldsymbol{R}}{\partial \phi}, \frac{\partial \boldsymbol{R}}{\partial z}\right\}$ forms a basis set of orthogonal vectors. This is called the natural basis for the coordinate system. Normalize them to form an orthonormal basis (the physical basis).

13. For Cartesian Coordinates, show that the natural basis coincides with the physical basis. [**Hint:** Obtain the natural basis from the set, $\left\{\frac{\partial \boldsymbol{R}}{\partial x}, \frac{\partial \boldsymbol{R}}{\partial y}, \frac{\partial \boldsymbol{R}}{\partial z}\right\}$. The physical basis is the normalized natural basis.]

14. Show that the product of a symmetric object with an antisymmetric object equals zero. For example given that $a_{mn}, m, n = 1,2,3$ is antisymmetric, Show that $a_{mn}x^m x^n = 0$.

15. Noting that $e_{ijk}\sigma_{jk} = 0$ observe that e_{ijk} is perfectly antisymmetric. What does this tell about σ_{jk}?

16. Given that A_{mn} and B_{mn} are symmetric, Let $A_{mn}\ x^m x^n = B_{mn}\ x^m x^n$ for arbitrary values of $x^i, i = 1,2,3$**,** show that $A_{mn} = B_{mn}$ for all values of $\boldsymbol{m}, \boldsymbol{n}$

17. Given that $a_{ij} = B_i B_j,$ where B_1, B_2 and $B_3$are constants Calculate the determinant $|a_{ij}|$

18. If A_{ij} is symmetric and B_{ij} is antisymmetric, find the value of $C = A_{ij}B_{ij}$

19. Show that the second-order system T_{ij} can be expressed as the sum of a symmetric system and an anti-symmetric system. Find an expression for these.

20. Show that the decomposition of a \rho tensor into the symmetric and anti-symmetric parts is unique.

21. The angle $0 \leq \theta \leq \pi$ between two skew lines in space is defined as the angle between their direction vectors when these vectors are placed at the origin. Show that for two lines with direction numbers a_i and $b_i, i\ =\ 1;\ 2;\ 3$ the cosine of the angle between these lines satisfies

$$\cos\theta = \frac{a_i b_i}{\sqrt{(a_i a_i)}\sqrt{(b_i b_i)}}$$

22. Let $\lambda = A_{ij}x_i x_j$ where $A_{ij} = A_{ji}$**.** Calculate $(a)\,\frac{\partial\lambda}{\partial x_m}, (b)\,\frac{\partial^2\lambda}{\partial x_m \partial x_k}$

23. If $A_{ij} = A_i B_j \neq 0\ \forall\ i, j$ values and $A_{ij} = A_{ji}$ for $i, j = 1,2, \dots, N$ Show that $A_{ij} = \lambda B_i B_j$ where λ is constant. Find λ.

24. Let $x_i = a_{ij}\bar{x}_j \;\; i, j = 1,2,3.$ denote a change of variables from a barred system of coordinates to an unbarred system and assume that $A_i = a_{ij}A_j$ where a_{ij} are constants. $\bar{A}_{\boldsymbol{i}}$ is a function of the $\bar{x}_j$ variables and A_j is a function of the x_i variables. Calculate $\frac{\partial \bar{A}_i}{\partial \bar{x}_m}$

25. Show that, $e_{rst}e_{ijk} = \begin{vmatrix} \delta_{ri} & \delta_{rj} & \delta_{rk} \\ \delta_{si} & \delta_{sj} & \delta_{sk} \\ \delta_{ti} & \delta_{tj} & \delta_{tk} \end{vmatrix}$

26. The trilinear mapping, $[.,.,.]: \mathscr{V} \times \mathscr{V} \times \mathscr{V} \to \mathscr{R}$ from the product set $\mathscr{V} \times \mathscr{V} \times \mathscr{V}$ to real space is defined by: $[\mathbf{a}, \mathbf{b}, \mathbf{c}] \equiv \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$. Show that $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = [\mathbf{b}, \mathbf{c}, \mathbf{a}] = [\mathbf{c}, \mathbf{a}, \mathbf{b}] = -[\mathbf{b}, \mathbf{a}, \mathbf{c}] = -[\mathbf{c}, \mathbf{b}, \mathbf{a}] = -[\mathbf{a}, \mathbf{c}, \mathbf{b}]$

27. Given that, $[\mathbf{a}, \mathbf{b}, \mathbf{c}] \equiv \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$. Show that this product vanishes if the vectors $(\mathbf{a}, \mathbf{b}, \mathbf{c})$ are linearly dependent.

28. Show that the product of a symmetric and an antisymmetric object vanishes.

29. Show that the product $\mathbf{AA} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \mathbf{B}$ Can be written in indicial notation as, $a_{ij}a_{jk} = b_{ik}$.

30. Show that the cross product of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ in Cartesian general coordinates is $e_{ijk}a_ib_j\mathbf{e}_k$ where a_i, b_j are the respective components.