On Saturday, August 9, 2014, Taiwo Atinsola sent in this question:

Sir, while I was reading through the worked examples on Tensor calculus. You made the following simplification:

$$\frac{\partial T_{\alpha}^{\alpha}}{\partial T_{j}^{i}} = \delta_{\alpha}^{j}\delta_{i}^{\alpha}$$

I would like to know if there is an explanation to this or it is an identity that should be taken without question. So far, all other explanations have been very clear

oafak Replies:

We begin by observing the fact that T_j^i are nine independent variables that are components of the tensor in the expansion $\mathbf{T} = T^i_{.j}\mathbf{g}_i \otimes \mathbf{g}^j$. When therefore you have the regular partial derivative (I must emphasize that what you are dealing with here is not a Frechét derivative but a regular well-known partial derivative – no new ideas here!), in order to know the correct answer, recall that these indices represent actual constant values. To be specific, suppose that $i = 1$, and $j = 1$. Recall that the αs here, because they are repeated, indicate a summation. Hence removing the mask completely,

$$\frac{\partial T_\alpha^\alpha}{\partial T_j^i} = \frac{\partial (T_1^1 + T_2^2 + T_3^3)}{\partial T_1^1} = 1 \; (= \delta_\alpha^1 \delta_1^\alpha)$$

Now, let $i = 1$, and $j = 2$. The expression now becomes,

$$\frac{\partial T_\alpha^\alpha}{\partial T_j^i} = \frac{\partial}{\partial T_2^1}(T_1^1 + T_2^2 + T_3^3) = 0 \; (= \delta_\alpha^1 \delta_2^\alpha)$$

Because the variable below does not appear explicitly anywhere in the sum above! It follows from here that, the only way a non-zero comes out is if both the upper index matches in the term above and the term below while the lower indices ALSO match! The first condition is δ_i^α while the second condition that must be concurrently fulfilled is δ_α^j. I am sure it is not difficult for you now to see, following the same argument, that,

$$\frac{\partial T_\beta^\alpha}{\partial T_j^i} = \delta_\beta^j \delta_i^\alpha$$

Which is even more general than the present example. We note lastly that this operation is valid only when the tensor is symmetrical. This is because in that case, there is no distinction between the components $T_{.j}^{i.}$ and $T_{j.}^{.i}$