

Good morning sir, I was actually in class yesterday and I guess I didn't post my question properly on your web page, that's why I am sending it to your mail (sorry for flouting your earlier instruction of not sending you personal mails)

**Problem:** Tensor Algebra pg 50-51 (second invariant), after contracting the equation below, what I got was different from what was in the slide and I really don't understand the "substitution"

$$\epsilon_{ijk} \left( T_{\beta}^{\alpha} T_{\alpha}^{\beta} \epsilon_{\alpha\beta k} + T_j^{\beta} T_k^{\gamma} \epsilon_{i\beta\gamma} + T_i^{\alpha} T_k^{\gamma} \epsilon_{\alpha j\gamma} \right)$$

This is what I got below.

$$T_{\alpha}^{\alpha} T_{\beta}^{\beta} - T_{\beta}^{\alpha} T_{\alpha}^{\beta} + T_{\beta}^{\beta} T_k^k - \boxed{T_{\beta}^{\beta} T_k^{\gamma}} + T_{\alpha}^{\alpha} T_k^k - T_{\gamma}^{\alpha} T_{\alpha}^{\gamma}$$

(sorry for having to make you re-post this question)

## oafak Replies

The actual working on my slides are as follows:

The last equality can be verified in the following way. Contracting the coefficient

$$\left( T_i^\alpha T_j^\beta \epsilon_{\alpha\beta k} + T_j^\beta T_k^\gamma \epsilon_{i\beta\gamma} + T_i^\alpha T_k^\gamma \epsilon_{\alpha j\gamma} \right)$$

with  $\epsilon^{ijk}$

$$\begin{aligned} & \epsilon^{ijk} \left( T_i^\alpha T_j^\beta \epsilon_{\alpha\beta k} + T_j^\beta T_k^\gamma \epsilon_{i\beta\gamma} + T_i^\alpha T_k^\gamma \epsilon_{\alpha j\gamma} \right) \\ &= \left( \delta_\alpha^i \delta_\beta^j - \delta_\alpha^j \delta_\beta^i \right) T_i^\alpha T_j^\beta + \left( \delta_\beta^j \delta_\gamma^k - \delta_\gamma^j \delta_\beta^k \right) T_j^\beta T_k^\gamma + \left( \delta_\alpha^i \delta_\gamma^k - \delta_\alpha^k \delta_\gamma^i \right) T_i^\alpha T_k^\gamma \\ &= T_\alpha^\alpha T_\beta^\beta - T_\beta^\alpha T_\alpha^\beta + T_\alpha^\alpha T_\beta^\beta - T_\beta^\alpha T_\alpha^\beta + T_\alpha^\alpha T_\beta^\beta - T_\beta^\alpha T_\alpha^\beta \\ &= 3 \left( T_\alpha^\alpha T_\beta^\beta - T_\beta^\alpha T_\alpha^\beta \right) \end{aligned}$$