oafak Replies

It is easily shown that the sixth-order Kronecker Delta can be written in terms of the second order Deltas in,

$$\delta^{i\beta\gamma}_{j\lambda\eta} = \epsilon^{i\beta\gamma}\epsilon_{j\lambda\eta} = \begin{bmatrix} \delta^i_j & \delta^i_\lambda & \delta^i_\eta \\ \delta^\beta_j & \delta^\beta_\lambda & \delta^\beta_\eta \\ \delta^\gamma_j & \delta^\gamma_\lambda & \delta^\gamma_\eta \end{bmatrix}$$

$$= \delta^i_j \begin{bmatrix} \delta^\beta_\lambda & \delta^\beta_\eta \\ \delta^\gamma_\lambda & \delta^\gamma_\eta \end{bmatrix} - \delta^i_\lambda \begin{bmatrix} \delta^\beta_j & \delta^\beta_\eta \\ \delta^\gamma_j & \delta^\gamma_\eta \end{bmatrix} + \delta^i_\eta \begin{bmatrix} \delta^\beta_j & \delta^\beta_\lambda \\ \delta^\gamma_j & \delta^\gamma_\lambda \end{bmatrix}$$

Contracting by allowing $i = \eta$, we have,

$$\delta^{i\beta\gamma}_{j\lambda i} = \epsilon^{i\beta\gamma}\epsilon_{j\lambda i} =$$

$$= \delta^i_j \begin{bmatrix} \delta^\beta_\lambda & \delta^\beta_i \\ \delta^\gamma_\lambda & \delta^\gamma_i \end{bmatrix} - \delta^i_\lambda \begin{bmatrix} \delta^\beta_j & \delta^\beta_i \\ \delta^\gamma_j & \delta^\gamma_i \end{bmatrix} + \delta^i_i \begin{bmatrix} \delta^\beta_j & \delta^\beta_\lambda \\ \delta^\gamma_j & \delta^\gamma_\lambda \end{bmatrix}$$

$$= \begin{bmatrix} \delta^\beta_\lambda & \delta^\beta_j \\ \delta^\gamma_\lambda & \delta^\gamma_j \end{bmatrix} - \begin{bmatrix} \delta^\beta_j & \delta^\beta_\lambda \\ \delta^\gamma_j & \delta^\gamma_\lambda \end{bmatrix} + 3 \begin{bmatrix} \delta^\beta_j & \delta^\beta_\lambda \\ \delta^\gamma_j & \delta^\gamma_\lambda \end{bmatrix}$$

$$= - \begin{bmatrix} \delta^\beta_j & \delta^\beta_\lambda \\ \delta^\gamma_j & \delta^\gamma_\lambda \end{bmatrix} - \begin{bmatrix} \delta^\beta_j & \delta^\beta_\lambda \\ \delta^\gamma_j & \delta^\gamma_\lambda \end{bmatrix} + 3 \begin{bmatrix} \delta^\beta_j & \delta^\beta_\lambda \\ \delta^\gamma_j & \delta^\gamma_\lambda \end{bmatrix} = \begin{bmatrix} \delta^\beta_j & \delta^\beta_\lambda \\ \delta^\gamma_j & \delta^\gamma_\lambda \end{bmatrix}$$

$$= \delta^{\beta\gamma}_{j\lambda}$$

Contracting once again, let $\gamma = \lambda$, we easily see that

$$\delta^{\beta\gamma}_{j\lambda} = 2\delta^\beta_j$$