

James, you are correct that,

$$\mathbf{g}_j \times \mathbf{g}_k = \epsilon_{ijk} \mathbf{g}^i$$

First note that in the above expression, the index i is a dummy index and can be replaced by any unused index. For the sake of argument, let us replace it with m so that, $\mathbf{g}_j \times \mathbf{g}_k = \epsilon_{mjk} \mathbf{g}^m$. Now let us take the dot product of both sides of this equation with the base vector \mathbf{g}_i so that,

$$\begin{aligned} \mathbf{g}_i \cdot \mathbf{g}_j \times \mathbf{g}_k &= \mathbf{g}_i \cdot (\epsilon_{mjk} \mathbf{g}^m) \\ &= \epsilon_{mjk} \mathbf{g}_i \cdot \mathbf{g}^m \\ &= \epsilon_{mjk} \delta_i^m = \epsilon_{ijk} \end{aligned}$$

So that the scalar triple product of the coordinate bases yields the scalar component, ϵ_{ijk} of the alternating, third-order absolute tensor

$$\epsilon = \epsilon_{ijk} \mathbf{g}^i \otimes \mathbf{g}^j \otimes \mathbf{g}^k$$

The fact that this is an absolute tensor is beyond our discussion scope here.