By the definition of the scalar triple product, we find that,

$$
\begin{aligned}
[(\boldsymbol{T}\boldsymbol{a}),(\boldsymbol{T}\boldsymbol{b}),(\boldsymbol{T}\boldsymbol{c})] &= \epsilon_{ijk} T^i_\alpha a^\alpha T^j_\beta b^\beta T^k_\gamma c^\gamma \\
&= \epsilon_{ijk} T^i_\alpha T^j_\beta T^k_\gamma a^\alpha b^\beta c^\gamma \\
&= e_{ijk} T^i_1 T^j_2 T^k_3 \epsilon_{\alpha\beta\gamma} a^\alpha b^\beta c^\gamma \\
&= \det(\boldsymbol{T})[\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}].
\end{aligned}
$$

Using the definition of the third invariant. Consequently,

$$I_3(\boldsymbol{T}) = \det(\boldsymbol{T}) = e_{ijk} T^i_1 T^j_2 T^k_3$$

We can also express this using similar arguments as,

$$I_3(\boldsymbol{T}) = \det(\boldsymbol{T}) = e^{ijk} T^1_i T^2_j T^3_k$$

We here establish the equality assumed above that,

$$\epsilon_{ijk} T^i_\alpha T^j_\beta T^k_\gamma = \epsilon_{ijk} T^i_1 T^j_2 T^k_3 e_{\alpha\beta\gamma}$$

We do this by first establishing the fact that the LHS is completely antisymmetric in α, β and γ. We first note that the indices i, j and k are dummy and therefore,

$$\epsilon_{ijk} T^i_\alpha T^j_\beta T^k_\gamma = \epsilon_{kji} T^k_\alpha T^j_\beta T^i_\gamma = \epsilon_{kji} T^i_\gamma T^k_\alpha T^j_\beta = -\epsilon_{ijk} T^i_\gamma T^j_\beta T^k_\alpha$$

Hence we see immediately that a simple swap of α and γ changes sign just as any other two lower symbols. Also note that the both sides take the same values when α, β and γ take the values of 1,2 and 3. The swapping of the indices α, β and γ make this value positive or negative in the same antisymmetric way. This completes the proof of equality.