

Consider the Tensor in component form:

$$[T_{ij}] = \begin{pmatrix} 9 & 1 & 2 \\ 3 & 8 & 4 \\ 5 & 6 & 7 \end{pmatrix}$$

Use Mathematica to find the characteristic polynomial,

$$\lambda^3 - I_1\lambda^2 + I_2\lambda - I_3$$

by simply doing the following: After invoking *Mathematica*, type,

1. `T={{9,1,2},{3,8,4},{5,6,7}}`
2. `CharacteristicPolynomial[T,λ]`

Which immediately gives you the polynomial ,

$$\lambda^3 - 24\lambda^2 + 154\lambda - 243 = 0$$

From which you can see that all the invariants of the tensor. We may now invoke the Cayley-Hamilton and write that

$$\mathbf{T}^3 - 24\mathbf{T}^2 + 154\mathbf{T} - 243\mathbf{1} = \mathbf{0}$$

since the tensor must satisfy its own characteristic equation. Pre-multiplying by \mathbf{T}^{-1} and rearranging this equation, we can write,

$$\begin{aligned}
\mathbf{T}^{-1} &= \frac{1}{I_3} (\mathbf{T}^2 - I_1 \mathbf{T} + I_2 \mathbf{1}) \\
&= \frac{1}{243} (\mathbf{T}^2 - 24\mathbf{T} + 154\mathbf{1}) \\
&= \frac{1}{243} (\text{MatrixPower}[\mathbf{T}, 2] - 24 \mathbf{T} + 154 \text{IdentityMatrix}[3])
\end{aligned}$$

This is an alternative way of calculating the inverse of this tensor. The result can be checked by a direct calculation of the inverse using the command, `Inverse[T]`. As a tricky follow-up, consider the following Mathematica code:

```

T = {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}
CharacteristicPolynomial[T, λ]
18 λ + 15 λ2 - λ3

```

Can you repeat what we just did above? Why or why not? Tell me on Tuesday