

Your question again demonstrates that you have not fully understood the substitution nature of the Kronecker Delta. It is a simple concept derived from the definition of the Delta:

$$g_{i\alpha} g^{k\beta} \delta_{\beta}^{\alpha} = g_{i1} g^{k\beta} \delta_{\beta}^1 + g_{i2} g^{k\beta} \delta_{\beta}^2 + g_{i3} g^{k\beta} \delta_{\beta}^3$$

We have simply recognized that α is a repeated index. On each term, we can further recognize the fact that β is a repeated index and obtain,

$$\begin{aligned} g_{i1} g^{k\beta} \delta_{\beta}^1 &= g_{i1} g^{k1} \delta_1^1 + \boxed{g_{i1} g^{k2} \delta_2^1} + \boxed{g_{i1} g^{k3} \delta_3^1} \\ g_{i2} g^{k\beta} \delta_{\beta}^2 &= \boxed{g_{i2} g^{k1} \delta_1^2} + g_{i2} g^{k2} \delta_2^2 + \boxed{g_{i2} g^{k3} \delta_3^2} \\ g_{i3} g^{k\beta} \delta_{\beta}^3 &= \boxed{g_{i3} g^{k1} \delta_1^3} + \boxed{g_{i3} g^{k2} \delta_2^3} + g_{i3} g^{k3} \delta_3^3 \end{aligned}$$

Each of the terms in the boxes vanish because the indices of their Kronecker Deltas are not equal. In the others, the Kronecker Deltas each has the value of unity.

Consequently,

$$\begin{aligned} g_{i\alpha} g^{k\beta} \delta_{\beta}^{\alpha} &= g_{i1} g^{k1} + g_{i2} g^{k2} + g_{i3} g^{k3} \\ &= g_{i\alpha} g^{k\alpha} = g_{i\beta} g^{k\beta} \end{aligned}$$

If, in the beginning, you simply realize that the Kronecker delta simply substitutes one of its symbols once the other is repeated in a multiplying term, then you will get the answer to this seemingly tedious problem in one step. Once you reached that level, then you have made some progress.

You can arrive at this tedious result in a single step by simply observing the substitution property of the Kronecker Delta:

$$g_{i\alpha} g^{k\beta} \delta_{\beta}^{\alpha}$$

Once β index is repeated as we have it here, we then substitute the α for k to obtain

$$g_{i\alpha} g^{k\beta} \delta_{\beta}^{\alpha} = g_{i\alpha} g^{k\alpha}$$

Look at it another way; imagine we rewrite it like this:

$$g_{i\alpha} \delta_{\beta}^{\alpha} g^{k\beta}$$

again, as before the repeated values of α can go and the remaining β on the delta will be substituted to obtain,

$$g_{i\alpha} g^{k\beta} \delta_{\beta}^{\alpha} = g_{i\beta} g^{k\beta}$$

Which is exactly the same as before as β here is a dummy index.

Now look at δ_{α}^{α} . Here again, the indices are repeated, hence there is an implied summation.

$$\delta_{\alpha}^{\alpha} = \delta_1^1 + \delta_2^2 + \delta_3^3$$

Each term is obviously unity hence the sum is 3.